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Kahn–Kalai conjecture

The **Kahn–Kalai conjecture**, also known as the **expectation threshold conjecture** or more recently the **Park-Pham Theorem**, was a conjecture in the field of graph theory and statistical mechanics, proposed by Jeff Kahn and Gil Kalai in 2006.[1][2] It was proven in a paper published in 2024.[3]

Background

This conjecture concerns the general problem of estimating when phase transitions occur in systems.[1] For example, in a random network with N nodes, where each edge is included with probability p, it is unlikely for the graph to contain a Hamiltonian cycle if p is less than a threshold value $(\log N)/N$, but highly likely if p exceeds that threshold.[4]

Threshold values are often difficult to calculate, but a lower bound for the threshold, the "expectation threshold", is generally easier to calculate.[1] The Kahn–Kalai conjecture is that the two values are generally close together in a precisely defined way, namely that there is a universal constant K for which the ratio between the two is less than $K \log l(\mathcal{F})$ where $l(\mathcal{F})$ is the size of a largest minimal element of an increasing family $\mathcal{F}$ of subsets of a power set.[3]

Proof

Jinyoung Park and Huy Tuan Pham announced a proof of the conjecture in 2022; it was published in 2024.[4][3]

References

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2. Kahn, Jeff; Kalai, Gil (2006-04-02). "Thresholds and expectation thresholds". arXiv:math/0603218 (https://arxiv.org/abs/math/0603218).
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4. Cepelewicz, Jordana (2022-04-25). "Elegant Six-Page Proof Reveals the Emergence of Random Structure" (https://www.quantamagazine.org/elegant-six-page-proof-reveals-the-emergence-of-random-structure-20220425/). *Quanta Magazine*. Retrieved 2022-04-25.

See also

- Percolation theory

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